Stability and The Existence of Coherent Structure in Demixed State of Binary BEC

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From a linear stability analysis of the Gross Pitaevskii equation for binary Bose Einstein condensates, it is found that the uniform state becomes unstable to a periodic perturbation of wave number k if k exceeds a critical value k_c. However we find that a stationary spatially periodic state does not exist. We show the existence of pulse type solutions, when the pulse structure for one condensate is strongly influenced by the presence of the other condensate.

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After the realization of Bose Einstein condensation (BEC) in dilute atomic gases of Rb [1, 2], Na [3], Li [4], there has been a lot of interest in the field of binary mixtures of the condensates. The physics of trapped dilute condensates which are inhomogeneous, finite sized systems has become one of the most exciting fields of research in the last decade. The first two component condensate was produced with different hyperfine states of Rb^{87}. The experiments on Rb^{87} atoms occupying the hyperfine states $|F = 1, m_F = -1\rangle$ and $|F = 2, m_f = 1\rangle$ have been done by JILA group(Myatt et al.,1997) [2]. Later the group in MIT has been able to trap atoms in F=1 with different m_F values using optical methods. The repulsive interaction between the hyperfine states was reported and since then the stability of binary Bose-Einstein Condensates [5, 6] has been extensively studied. Specially in case of two species BEC there is type of new phenomenology associated with it. Lots of theoretical and experimental works have been done on Na- Rb mixture [7] and on K-Rb mixtures [11, 12]. Infact there is a range of interspecies interaction strength in which a Na-Rb mixture can be stable in harmonic trap [8]. In case of demixed state of binary BEC there is a tendency of spatial separation of the condensates, the interatomic interaction playing the vital role. The possibility of phase separation of the components and the density patterns of the system has been theoretically reported [9, 10]. The spatial separation between the condensates has been verified experimentally by the JILA experiment [13]. Recently the miscibility and immiscibility of the dual species (Rb^{85} and Rb^{87}) BEC depending on the strength of the external magnetic field and hence on the scattering length has been reported experimentally[14]. In this paper we are able to show that under the condition $\Delta < 0$ (the same condition as experimentally observed[14])the phase separation occurs in dual species BEC and there is localization of one species in presence of other in a pulse structure, where $\Delta = \frac{a_0 a_1}{a_{01}^2} - 1$. We also have depicted how the density of one species affects the localization of other species.

In many physical systems the spreading of wavepackets occurs in the dispersive(linear) medium because different component of wavepackets have different velocities. It is the speciality of nonlinear systems that there exist some coherent structures [15]. They are the special type of solutions of pattern forming system. Pulse, front, source and sink are the four basic things to deal with in the study of coherent structures. Several solutions like domain walls [16], antidark or grey solitons of one component bound to a bright or dark solitons [17] in the other, have been found for two component Gross Pitaevskii equation. A particularly interesting variant involves introducing a linear coupling between the two condensates. In the case of this coupling being given a time dependence, it has been shown that one can induce a Rabi switch between the two condensates. The pulse type solutions are extremely important as they behave like the solitons and propagate within the medium keeping their shape invariant. So if there exists a pulse structure, the multicomponent system can have a species wise localization. We have found both analytically and numerically pulse type solutions satisfying Gross Pitaevskii (GP) equation.

When the atoms are cooled to extremely low temperatures the de Broglie wave length associated with the atoms starts to increase. At temperatures below T_c (critical temperature) the wave functions start to overlap. Under these conditions the atoms start to condense in a single quantum state which is known to be the Bose Einstein condensation. The nonlinear interaction between the atoms in this regime is due to the s-wave scattering between the atoms and the interactions are described by the single parameter a, called the s-wave scattering length as the interaction takes place at a very low kinetic energy. The interaction plays a vital role. For a given atomic species under attractive interaction a collapse may occur if the particle number in the condensate exceeds a certain critical value N_c i.e, at high density

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state though for confined atomic gases the BEC is realized and also the collapsing dynamics has been observed [Gerton, et al. 2000]. Most experiments have been done with the atoms in repulsive interatomic interaction where the Thomas-Fermi approximation holds good. Here we restrict ourselves to a repulsive interaction.

The GP equation for describing the binary mixture is written in terms of two fields $\psi_0(\mathbf{r},t)$ and $\psi_1(\mathbf{r},t)$ which are complex valued functions representating the expectation values of the corresponding quantum fields. In absence of any external potential coupled GP equation has the following well known form,

$$i\hbar\partial_t\psi_0 = \Big(-\frac{\hbar^2}{2m}\nabla^2 + g_0|\psi_0|^2 + g_{01}|\psi_1|^2\Big)\psi_0 \tag{1}$$

$$i\hbar\partial_t\psi_1 = \Big(-\frac{\hbar^2}{2m}\nabla^2 + \epsilon + g_1|\psi_1|^2 + g_{01}|\psi_0|^2\Big)\psi_1 \tag{2}$$

where,

$$g_0 = \frac{4\pi\hbar^2 a_0}{m}, g_1 = \frac{4\pi\hbar^2 a_1}{m} \text{ and } g_{01} = \frac{4\pi\hbar^2 a_{01}}{m}$$

a_0 and a_1 are the scattering length of the particles belonging to state 0 and state 1. a_{01} is determined when an atom in the state 0 is scattered by the atom in the state 1. The validity of GP equation depends on the fact that s-wave scattering length is much smaller than the average interparticle separation.

A spatially uniform solution has the form:

$$\psi_j = e^{-\frac{i\mu_j t}{\hbar}}\frac{1}{\sqrt{V}} \qquad ; j = 0,1 \tag{3}$$

The chemical potentials satisfy,

$$\begin{aligned}\mu_0 &= g_0|\psi_0|^2 + g_{01}|\psi_1|^2 = (g_0+g_{01})/V\\ \mu_1 &= \epsilon + g_1|\psi_1|^2 + g_{01}|\psi_0|^2 = (\epsilon + g_0 + g_{01})/V\end{aligned} \tag{4}$$

To test the stability of the uniform state, we introduce perturbation.

$$\left.\begin{aligned}\delta\psi_j &= A_j e^{-\frac{i\mu_j t}{\hbar}} e^{i(\mathbf{k}.\mathbf{r}-\omega t)}\\ \delta\psi_j^* &= A_j' e^{\frac{i\mu_j t}{\hbar}} e^{i(\mathbf{k}.\mathbf{r}-\omega t)}\end{aligned}\right\} \qquad j = 0,1 \tag{5}$$

Linearizing Eqs. (1) and (2) in $\partial\psi_j$ and its complex conjugates, we get

$$\begin{pmatrix} \tilde{K} - \frac{g_0}{V} & -\frac{g_{01}}{V} & -\frac{g_0}{V} & -\frac{g_{01}}{V}\\ \frac{g_{01}}{V} & \tilde{K} - \frac{g_1}{V} & -\frac{g_{01}}{V} & -\frac{g_1}{V}\\ -\frac{g_0}{V} & -\frac{g_{01}}{V} & \tilde{K} - \frac{g_0}{V} & -\frac{g_{01}}{V}\\ -\frac{g_{01}}{V} & -\frac{g_1}{V} & -\frac{g_{01}}{V} & \tilde{K} - \frac{g_1}{V}\end{pmatrix} \times \begin{pmatrix} A_0\\ A_1\\ A_0'\\ A_1'\end{pmatrix} = 0$$

Where,

$$\begin{aligned}\tilde{K} &= \omega\hbar - K\\ K &= \frac{k^2\hbar^2}{2m}\end{aligned}$$

Consistency requires,

$$\begin{aligned}\omega^2 = {} & \frac{K^2}{\hbar^2} + (g_0+g_1)\frac{K}{\hbar^2 V}\\ & \pm \frac{K}{\hbar^2 V}\sqrt{g_0^2 + g_1^2 - 2g_0g_1 + 4g_{01}^2}\end{aligned} \tag{6}$$

If ω^2 turns out to be real then the mixture will be stable for all k. Hence we have to analyze whether ω^2 is real or not. It can be found from Eq. (6) that for all values of k the mixture is not stable. The mixture is stable only for those values of k for which

$$K^2 + \frac{2K}{V}(g_0+g_1) < (g_{01}^2 - g_0g_1) \tag{7}$$

So, by taking constant ϕ solution the stability of the mixture is not possible for all values of k. Thus we need to Analise the solution having spatially dependent terms, to see if there is a spatially periodic stationary state.

Accordingly, we try the stationary spatially periodic solutions,

$$\psi_{0,1} = \Big[\frac{1}{\sqrt{V}} + \delta\phi_{0,1}(x)\Big]e^{-\frac{i\mu t}{\hbar}} \tag{8}$$

Where $\partial\phi_{0,1}(x)$ are spatially periodic functions having the expansions,

$$\begin{aligned}\delta\phi_0(x) &= \sum A_n\cos(nkx)\\ \delta\phi_1(x) &= \sum B_n\cos(nkx)\end{aligned}$$

It should be clear that the wavenumber k would be found from Eq. (6) with $\omega = 0$ and thus immediately leads to

$$\frac{\hbar^2 k^2}{2m} = -\frac{g_0+g_1}{V} \pm \sqrt{(g_0+g_1)^2 - 4g_0g_1 + 4g_{01}^2} \tag{9}$$

This can never yield two positive values for k^2 and hence does not support a stationary spatially periodic state. Consequently we seek a solution which is both space and time dependent and is uniformly propagating.

We have taken the coupling constants to be +ve implying a repulsive interaction between the atoms. But there may be a regime where the strength of repulsive interactions are too high to have the spatial inhomogeneity within the system. This results in the demixing instability within the system. Condensate 0 and 1 can separate spatially. It is found previously that spatially uniform condensate is stable when $g_{01} \le (g_0g_1)^{1/2}$. But when this condition is not satisfied there is a tendency of spatial separation of the condensates and the resulting state is called the demixed state. The tendency of separation of the condensates has been found in experiment by JILA group [13].

Now we investigate the possible structure for such a demixed state. For this we have taken,

$$\psi_j = e^{-\frac{i\mu_j t}{\hbar}} f_j(x-vt) \qquad \text{for j= 0,1} \tag{10}$$

with

$$f_0(\xi) = A(\xi)e^{i\phi_0(\xi)} \quad (11)$$
$$f_1(\xi) = B(\xi)e^{i\phi_1(\xi)} \quad (12)$$

where $(x - vt) = \xi$. i.e, we are considering two condensates moving with same velocity and in same direction. Inserting the above equations in Eq. (1) and (2),the flow equations for the binary system becomes,

$$\partial_\xi A = \kappa_0 A \quad (13)$$
$$\begin{aligned}\partial_\xi \kappa_0 &= K \\ &= -\kappa_0^2 - q_0^2 - \frac{2m}{\hbar^2}\mu_0 - \frac{2m}{\hbar}q_0 v \\ &\quad + \frac{2m}{\hbar^2}(g_0|A|^2 + g_{01}|B|^2)\end{aligned} \quad (14)$$
$$\partial_\xi q_0 = Q = \frac{2m}{\hbar}\kappa_0 v - 2\kappa_0 q_0 \quad (15)$$
$$\partial_\xi B = \kappa_1 B \quad (16)$$
$$\begin{aligned}\partial_\xi \kappa_1 &= \bar{K} \\ &= -\kappa_1^2 - q_1^2 - \frac{2m}{\hbar^2}\mu_1 - \frac{2m}{\hbar}q_1 v \\ &\quad + \frac{2m}{\hbar^2}(g_1|B|^2 + g_{01}|A|^2)\end{aligned} \quad (17)$$
$$\partial_\xi q_1 = \bar{Q} = \frac{2m}{\hbar}\kappa_1 v - 2\kappa_1 q_1 \quad (18)$$

where, $\partial_\xi \phi_j = q_j$ for j=0,1 and $\kappa_0 = \frac{1}{A}\partial_\xi A$, $\kappa_1 = \frac{1}{B}\partial_\xi B$. Inserting the value of κ_0 in Eq. (15) we get the value of $q_0 = \frac{mv}{\hbar}$ and from Eq. (14)

$$\frac{\partial^2}{\partial \xi^2} = \gamma_0 A \quad (19)$$

Where,

$$\begin{aligned}\gamma_0(A^2) = q_0^2 - \frac{2m}{\hbar^2}\mu_0 - \frac{2m}{\hbar}q_0 v \\ + \frac{2m}{\hbar^2}(g_0|A|^2 + g_{01}|B|^2)\end{aligned} \quad (20)$$

After a little bit calculation from Eq. (19), the relation comes out to be

$$\kappa_0^2 = \frac{1}{A^2}\int_0^{A^2} \gamma_0(y)dy \quad (21)$$
$$= r_0 + r_1 A^2 \quad (22)$$

Where,

$$r_0 = \frac{m}{\hbar^2}[-2\mu_0 - mv^2 + 2g_{01}B^2] \quad (23)$$
$$r_1 = \frac{m}{\hbar^2}g_0 \quad (24)$$

Now we can write the pulse type solution for condensate 0 which will be of the form of,

$$A^2 = (M_1 + M_2 \cosh M_3\xi)^{-1} \quad (25)$$

with

$$M_1 = -\frac{r_1}{2r_0} \quad (26)$$
$$M_2^2 = \frac{r_1^2}{4r_0^2} \quad (27)$$
$$M_3^2 = 4r_0 \quad (28)$$

The structure of this pulse of one condensate thus depends on the amplitude of the other pulse at that location and at that time. As the pulses are like the solitons of the system so they will propagate keeping their shape constant for long time. Hence both of the condensates will localize and the localization will be gaussian in shape.

A careful observation of Eq. (23) reveals the fact that there is a competition among the terms. Since the second term in the parenthesis of the expression of r_0 is the kinetic energy of the atoms, we can neglect it compared to others in the low temperature regime. Hence r_0 comes out to be $(2g_{01}B^2 - 2\mu_0)$. Now three cases may happen according to the competition between the terms.

Case 1: When $\mu_0 = g_{01}B^2$, M_1 and M_2 turn out to be ∞, i.e, according to the Eq. (25) A^2 is zero. So the pulse structure vanishes.

Case 2: When $\mu_0 < g_{01}B^2$, M_1 is -ve. Since $M_2 = \pm M_1$, for $M_1 < 0$ and $M_2 < 0$, A^2 turns put to be -ve which can't happen and for $M_1 < 0$ and $M_2 > 0$ there is a infinite divergence at $\xi = 0$.

Case 3: Only for the remaining condition $\mu_0 > g_{01}B^2$ the pulse structure exist. Now we will express the condition in more well known form which is the evidence of the spatial separation of the two species being pulse structured.

For the condensate 0 with density n_0 and condensate 1 with density n_1

$$g_{01}B^2 < \mu_0 = \frac{4\pi\hbar^2 a_0 n_0}{m_0} = g_0 n_0 \quad (29)$$
$$g_{01}A^2 < \mu_1 = \frac{4\pi\hbar^2 a_1 n_1}{m_1} = g_1 n_1 \quad (30)$$

Now the order parameter $\psi_0 = \sqrt{N_0}\phi_0$ and $\psi_1 = \sqrt{N_1}\phi_1$. For the constant ϕ solution i.e., for $\phi = \frac{1}{\sqrt{V}}$, $|\psi_0|^2 = n_0 = |A|^2 = A^2$ (A being real) and $|\psi_1|^2 = n_1 = |B|^2 = B^2$(B being real). Multiplying Eqs. (29) and (30) now the condition becomes $g_{01}^2 < g_0 g_1$ which implies $\Delta < 0$, i.e., the condition for which immiscibility between the species [14] because of strong interspecies interaction occurs and hence there exists the pulse type localization of each species.

Scaling $B' = 1 - 8\pi \times 10^{-21}B^2$ and $\tilde{B} = 2 \times 10^6 B'$ we have plotted the pulse structure (Eq. (25)) for different values of $\tilde{B}$ which is the scaled version of B. In figures $\tilde{A}$ and $\tilde{B}$ is represented by A and B.

Fig. 1 depicts the structure of the pulse of condensate 0 changing with the amplitude of condensate 1. As

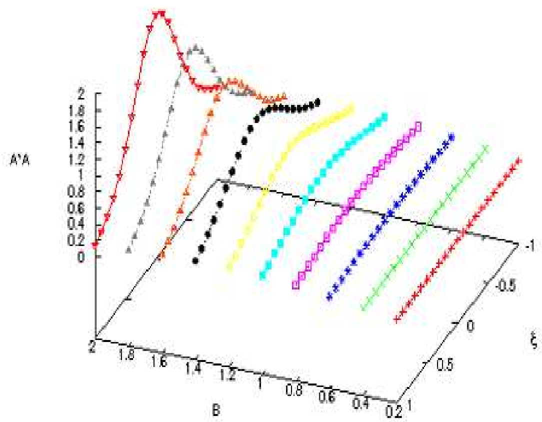

FIG. 1: Sensitivity of the pulse structure on the values of B. Taking, m=23 amu., $v \sim 10^{-7}$ m/s, $\mu \sim K_B T$, T=100 nK, $a_0, a_{01} \sim 10^{-9}$ m. We have plotted $\tilde{A}^2$ with different values of $\tilde{B}$, $\tilde{A} = \sqrt{80\pi} \times 10^{-5} A$.

the value of B increases the structure of the pulse becomes more and more steep i.e, when the amplitude of condensate 1 is high it will make the atoms of the other condensate to localize in space while when the amplitude of condensate 1 is low the atoms of condensate 0 has enough space to spread over. So the localization of one condensate is greatly affected by the presence of another condensate.

In Fig. 2 it is obvious that at the lower value of B i.e, at B=0.2, 0.6 the atoms of condensate 0 spreads over the space. When the value of B increases the spreading decreases and the atoms get gradually localized. At B=1.8, 2.2 the localization is very prominent. Now instead of condensate 0 if we start with the GP equation for condensate 1 the pulse would generate for condensate 1 and the qualitative nature of the pulse for condensate 1 will remain the same.

So far we are dealing with the condensates having the same velocity and moving in the same direction. Now we will investigate the situation when two condensates having different velocity propagates in opposite direction. We take condensate 0 and 1 have the velocity v_0 and v_1 respectively and they are going in opposite direction. Here we have not encountered the case (condensates approaching each other) of collision between the condensates. We will take the case when from a mixture of two condensates they are starting to go apart. So to deal with the case, we will take $\psi_0 = e^{-\frac{i\mu_0 t}{\hbar}} f_0(x - v_0 t)$ and $\psi_1 = e^{-\frac{i\mu_1 t}{\hbar}} f_1(x + v_1 t)$.

Where,

$$f_0(\xi_0) = A(\xi_0) e^{i\phi_0(\xi_0)} \tag{31}$$

$$f_1(\xi_1) = B(\xi_1) e^{i\phi_1(\xi_1)} \tag{32}$$

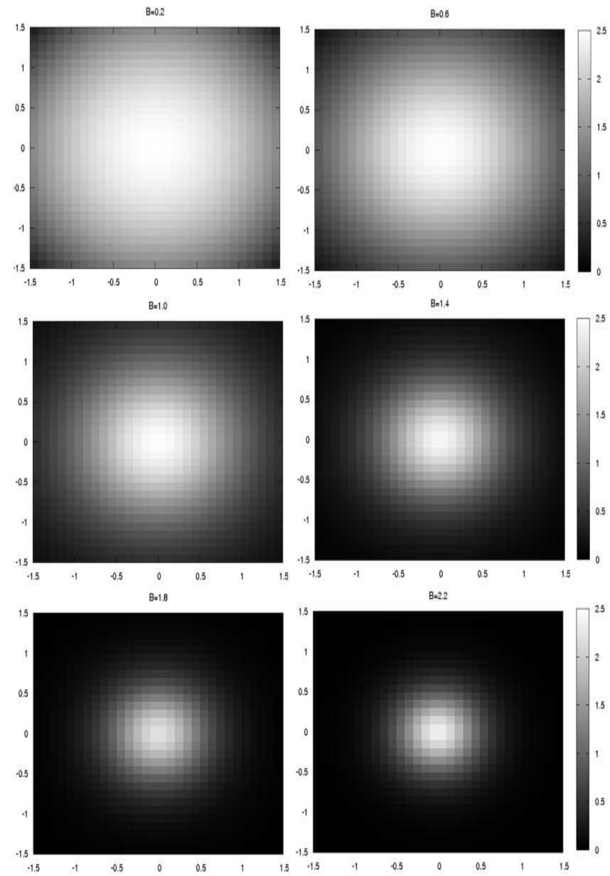

FIG. 2: 2D visualization of the pulse structure for different values of B. From upper left corner to lower right corner, the values of B are 0.2, 0.6, 1.0, 1.4, 1.8 and 2.2 respectively.

After doing little bit of calculation, we will find

$$r_0|_0 = \frac{m}{\hbar^2}[-2\mu_0 - m v_0^2 + 2g_{01}[B(\xi_1)]^2] \tag{33}$$

$$r_0|_1 = \frac{m}{\hbar^2}[-2\mu_1 - m v_1^2 + 2g_{01}[A(\xi_0)]^2] \tag{34}$$

In the expression for r_0 for both the condensates 2nd term is always +ve, where as 1st and 3rd term is always -ve. Hence for a particular value of v_0 and v_1 there will be the situation when r_0 will become zero and the pulse for the corresponding condensates will cease to exist. We have found that at $v_0^2 = \frac{2}{m}(g_{01}B^2 - \mu_0)$ the pulse for condensate 0 will not exist but that of condensate 1 remains and at $v_1^2 = \frac{2}{m}(g_{01}A^2 - \mu_1)$ pulse of condensate 1 vanishes though that of condensate 0 survives, v_0 and v_1 being of the order of 10^{-2} meter/sec.

We have studied the stability of binary BEC and we have found that for spatial independent solution, the system is not stable for all values of k. The system is stable only for those k which satisfy the condition $K^2 + \frac{2K}{V}(g_0 + g_1) < (g_{01}^2 - g_0 g_1)$. By taking uniformly propagating solution of GP equation. We have observed

the existence of pulse in the demixed state when the condition of immiscibility holds. Under such condition the density of one species affects the localization of others. When two condensates moving in the opposite direction, at the low velocity both of the pulses survive and at high velocity regime, at a particular velocity (either v_0 for condensate 0 and v_1 for condensate 1)one of the pulses ceases to exist.

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